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The onset of a Darcy–Brinkman convection using a thermal nonequilibrium model. Part II

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Abstract

A linearized analysis is performed in this paper in order to analyze the onset of Darcy–Brinkman convection in a fluid-saturated porous layer heated from below, by considering the case when the fluid and solid phases are not in local thermal equilibrium. The problem is transformed into an eigenvalue equation which is solved in a first step by using an one-term Galerkin approach: an explicit relationship between the Darcy–Rayleigh number based on the fluid properties R and the horizontal wave number k is obtained. Minimization of R over k is performed analytically and finally, critical values for R and k are obtained for various values of the three parameters of the problem, namely the Darcy number D, the porosity-scaled conductivity ratio γ and the scaled inter-phase heat transfer coefficient H. In a second step, a general N-terms Galerkin approach is used and finally comparisons are performed between the results given by these two approaches.

Keywords: Convection; Porous layer; Darcy–Brinkman formulation; Thermal nonequilibrium; Galerkin approach

1. Introduction

The onset of convection in a porous layer heated from below is the "porous medium" version of the Bénard problem in clear fluids. A review of the state-of-the art in this area of research may be found in Rees [1] and Nield and Bejan [2]. Earlier papers in this field are those by Homsy and Walker [3] and Combarnous [4].

On the other hand, in a porous medium the volume averaged temperatures of the solid and fluid phases are generally different from one another and this is termed as local thermal non-equilibrium. Nield and Bejan [2] have discussed in their book a two-field model for the energy equation. Many studies in the literature of the non-equilibrium effects concentrated on the forced convective flows, see Kuznetsov [5], but we are interested here in the combination of such effects with natural convective flows, see, for instance, the review by Rees and Pop [6]. Rees and Pop have investigated in a couple of papers [7–9] the thermal non-equilibrium effect on several free convective flows in fluid saturated porous media.

An earlier work in the field is that by Banu and Rees [10], who were able to determine the Rayleigh number corresponding to the onset of convection in the conditions of thermal nonequilibrium between fluid and solid phases of the porous layer. Related to the present discussion is also the paper by Rees [11], where the numerical study by Homsy and Walker [3] was extended, by performing an asymptotic analysis of the singular perturbation problem which arises in the small Darcy–Rayleigh number limit. Postelnicu and Rees [12] extended [10] by including the boundary effects as modeled by the Brinkman terms. They included also the form-drag, but it was shown that these terms have no effect on stability criteria, since the basic state whose stability is being analyzed was one of no flow.

Three recent papers dealing with the onset of convection in porous layers using a thermal nonequilibrium model are [13], [14] and [15]. Malashetty et al. [13] carried out a similar study as in [12], while Malashetty et al. [14] analyzed the effects of thermal nonequilibrium and anisotropy in both mechanical and thermal properties of the porous medium on the onset of convection. They performed both an analytical and asymptotic study within the framework of a linear stability anal-

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Nomenclature

b	form drag coefficient
c	specific heat . . . . . J/kg K
d	depth of the porous layer . . . . . m
D	Darcy number
F_1, F_2	dimensionless coefficients, (11)
f	y-dependent part of the perturbed stream function (dimensionless)
g	y-dependent part of the perturbed fluid phase temperature (dimensionless); also, acceleration due to gravity . . . . . m/s^2
h	y-dependent part of the perturbed solid phase temperature (dimensionless); also, inter-phase heat transfer coefficient . . . . . W/m^3 K
H	scaled inter-phase heat transfer coefficient
K	permeability . . . . . m^2
k	horizontal wave number; also, with subscript, thermal conductivity . . . . . W/m K
R	Darcy–Rayleigh number
V	velocity . . . . . m/s
T	temperature . . . . . K
x, y	Cartesian co-ordinates taken along the lower surface of the porous layer and normal to it, respectively

Greek letters

α	diffusivity ratio
β	coefficient of expansion . . . . . K^{-1}
ρ	density . . . . . kg/m^3
κ	diffusivity . . . . . m^2/s
ε	porosity
ψ	stream function, dimensionless
Ψ	perturbation of the stream function
θ	dimensionless temperature of the fluid phase in the basic conduction state
Θ	perturbation of the fluid phase non-dimensional temperature
ϕ	dimensionless temperature of the fluid phase in the basic conduction state
Φ	perturbation of the solid phase non-dimensional temperature
γ	porosity-scald conductivity ratio

Subscripts

c	cold
e	effective (refers to viscosity)
f	fluid
h	hot
s	solid

Superscripts

$'$	= differentiation with respect to y

ysis. Malashetty et al. [15] used a generalized Darcy model to analyze an Oldroyd-B fluid saturated porous medium and a two-field model for energy equation each representing solid and fluid phases separately. Linear stability analysis revealed that there is a competition between the processes of viscous relaxation and thermal diffusion causing the first convective instability to be oscillatory rather than stationary.

Postelnicu [16] dealt with the onset of mixed convection in a porous layer heated from below, by considering the effect of inertia, when the fluid and solid phases are not in local thermal equilibrium, in a frame of a linear stability analysis. The mixed convection is considered in the sense of a mean horizontal pressure gradient. Critical Rayleigh and wave numbers have been analytically determined in both two and three-dimensional cases.

The present paper continues the study by Postelnicu and Rees [12], named hereinafter Part 1, by taking into account isothermal rigid boundaries. In Part 1 the case of stress-free boundaries was considered, leading to the possibility of an analytical approach. In the present case, such an analytical tackling is no more possible, so that we have to use a numerical approach, based on the Galerkin technique, in order to find the critical Darcy–Rayleigh number and wave number at which the convection occurs.

2. Mathematical analysis

We consider a porous layer saturated with an incompressible Newtonian fluid. The layer is heated from below, its lower surface, located at $y = 0$, being held at a temperature T_h, while the upper one, located at $y = d$, is at $T_c < T_h$. The porous layer is isotropic, but the local thermal equilibrium does not apply.

Considering that both form-drag and boundary effects are significant and invoking the Boussinesq approximation, the governing equations read [2]

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\frac{\partial \mathbf{V}}{\partial t} + \frac{\rho_f}{\varepsilon^2}\mathbf{V}\cdot\nabla\mathbf{V} = -\nabla p + \mu_e \nabla^2 \mathbf{V} - \frac{\mu_f}{K}\mathbf{V} + \rho_f g \beta (T - T_c)\mathbf{y} - \frac{\rho_f b}{\sqrt{K}}\mathbf{V}|\mathbf{V}| \tag{2}$$

$$\varepsilon(\rho c)_f \frac{\partial T_f}{\partial t} + (\rho c)_f \mathbf{V}\cdot\nabla T_f = \varepsilon k_f \nabla^2 T_f + h(T_s - T_f) \tag{3}$$

$$(1-\varepsilon)(\rho c)_s \frac{\partial T_s}{\partial t} = (1-\varepsilon) k_s \nabla^2 T_s - h(T_s - T_f) \tag{4}$$

where usual notations are used. The boundary conditions are

$$\mathbf{V} = 0, \qquad T = T_h, \quad \text{at } y = 0 \tag{5a}$$

$$\mathbf{V} = 0, \qquad T = T_c, \quad \text{at } y = d \tag{5b}$$

We introduce the dimensionless quantities

$$\tilde{\mathbf{x}} = \frac{1}{d}\mathbf{x}, \qquad \tilde{t} = \frac{(\rho c)_f d^2}{k_f} t, \qquad \tilde{\mathbf{V}} = \frac{\varepsilon k_f}{(\rho c)_f d}\mathbf{V}$$

$$\tilde{p} = \frac{\mu k_f}{(\rho c)_f K} p, \qquad \theta = \frac{T_f - T_c}{T_h - T_c}, \qquad \phi = \frac{T_s - T_c}{T_h - T_c} \tag{6}$$

and the governing equations (1)–(4) become

$$\nabla \cdot \mathbf{V} = 0 \tag{7}$$

$$\varepsilon F_1 \frac{\partial \mathbf{V}}{\partial t} + F_1 \mathbf{V} \cdot \nabla \mathbf{V} = -\varepsilon^2 F_1 \nabla p + D\nabla^2 \mathbf{V} - \mathbf{V} + R\theta \mathbf{y} - F_2 \mathbf{V}|\mathbf{V}| \tag{8}$$

$$\frac{\partial \theta}{\partial t} + \mathbf{V} \cdot \nabla \theta = \nabla^2 \theta + H(\phi - \theta) \tag{9}$$

$$\alpha \frac{\partial \phi}{\partial t} = \nabla^2 \varphi + \gamma H(\theta - \phi) \tag{10}$$

where the tildes were omitted, for convenience of presentation. In Eqs. (8)–(10) the following notations were introduced

$$F_1 = \frac{\rho_f \kappa K}{\varepsilon^2 d^2 \mu_f}, \qquad F_2 = \frac{\rho_f \kappa K^{1/2}}{d\mu_f}, \qquad D = \frac{\mu_e}{\mu_f} \cdot \frac{K}{d^2}$$

$$H = \frac{hd^2}{\varepsilon k_f}, \qquad \alpha = \frac{(\rho c)_s}{(\rho c)_f} \cdot \frac{k_f}{k_s}, \qquad R = \frac{\rho_f g \beta (T_h - T_c) K d}{\varepsilon \mu_f k_f} \tag{11}$$

where R is the Darcy–Rayleigh number based on the fluid properties, D is the Darcy number, γ is the porosity-scaled conductivity ratio and H the scaled inter-phase heat transfer coefficient.

The boundary conditions (5) become

$$\mathbf{V} = 0, \qquad \theta = \phi = 1 \quad \text{on } y = 0 \tag{12a}$$

$$\mathbf{V} = 0, \qquad \theta = \phi = 0 \quad \text{on } y = 1 \tag{12b}$$

Focusing on the two-dimensional case, the basic conduction profile, whose stability is studied here is

$$\psi = 0, \qquad \theta = \phi = 1 - y \tag{13}$$

where ψ is the dimensionless streamfunction, θ and ϕ are dimensionless temperatures in the fluid and solid phase. The basic conduction profile (13) is perturbed by setting

$$\psi = \Psi(x, y), \qquad \theta = 1 - y + \Theta(x, y)$$

$$\phi = 1 - y + \Phi(x, y) \tag{14}$$

and the problem in perturbed quantities reads

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} - D\left(\frac{\partial^4 \Psi}{\partial x^4} + 2\frac{\partial^4 \Psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \Psi}{\partial y^4}\right) = R\frac{\partial \Theta}{\partial x} \tag{15}$$

$$\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} + \frac{\partial \Psi}{\partial x} + H(\Phi - \Theta) = 0 \tag{16}$$

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \gamma H(\Theta - \Phi) = 0 \tag{17}$$

[see Part 1]. We mention that (15) is the Darcy–Brinkman equation, while (16) and (17) are the energy equations in the fluid and solid phase, respectively.

The boundary conditions for the perturbed equations (15)–(17) are

$$\Psi = \frac{\partial \Psi}{\partial y} = \Theta = \Phi = 0 \quad \text{on } y = 0 \text{ and } y = 1 \tag{18}$$

which correspond to rigid isothermal boundaries (RB). To this end, we remark that Postelnicu and Rees [12] have solved the problem consisting of Eqs. (15)–(17) in the case of isothermal stress-free boundaries (SFB). These authors were able to get analytical solutions, supplementing their study with an asymptotic analysis for both small and large values of H.

Eqs. (15)–(17) admit solutions in the form

$$\Psi = f(y)\sin kx, \qquad \Theta = g(y)\cos kx$$

$$\Phi = h(y)\cos kx \tag{19}$$

where k is the horizontal wave number. By substituting (19) into Eqs. (15)–(17), we obtain

$$-D(f^{iv} - 2k^2 f'' + k^4 f) + (f'' - k^2 f) = -Rkg \tag{20}$$

$$g'' - (k^2 + H)g + kf + Hh = 0 \tag{21}$$

$$h'' - (k^2 + \gamma H)h + \gamma H g = 0 \tag{22}$$

while the boundary conditions (18) become

$$f = f' = g = h = 0 \quad \text{on } y = 0 \text{ and } y = 1 \tag{23}$$

We notice to this end that primes denote differentiation with respect to y.

3. Numerical analysis and results

In the present case, of isothermal rigid boundaries, analytic progress is no more possible, so that a numerical approach is necessary. A Galerkin technique will be used on the basis that this method has the advantage of dealing with many parameters very economically (see, for instance, Finalyson [17]).

3.1. One-term Galerkin approach

We take

$$f = Af_1, \qquad g = Bg_1, \qquad h = Ch_1 \tag{24}$$

where f_1, g_1 and h_1 are trial functions and A, B and C are constants. The form of the boundary conditions (23) allows us to take

$$f_1 = y^2(1-y)^2, \qquad g_1 = y(1-y), \qquad h_1 = y(1-y) \tag{25}$$

Multiplying (20) by f and integrating from 0 to 1, (21) by g and integrating from 0 to 1, and (22) by h and integrating from 0 to 1, we get after some algebra the following algebraic homogeneous system

$$-\left[\frac{4}{5}D + \frac{2}{105}(2k^2 D + 1) + \frac{1}{630}k^2(1 + k^2 D)\right]A + \frac{1}{140}RkB = 0 \tag{26}$$

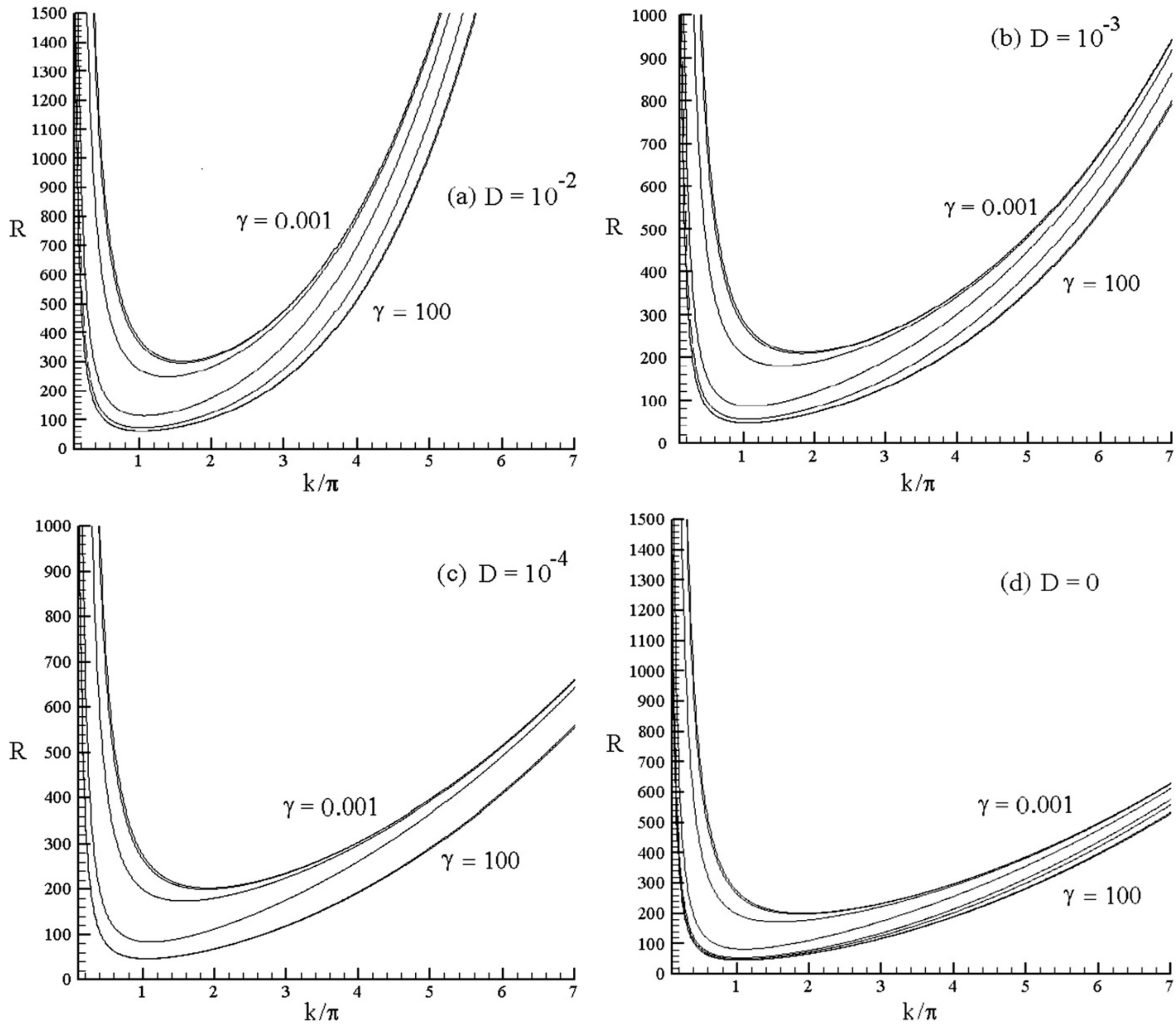

Fig. 1. Neutral curves for $H = 100$, $\gamma = 0.001, 0.01, 0.1, 1, 5, 10, 50$ and 100: (a) $D = 10^{-2}$, (b) $D = 10^{-3}$, (c) $D = 10^{-4}$, (d) $D = 0$.

$$\frac{1}{140}kA - \left[\frac{1}{3} + \frac{1}{30}(k^2 + H)\right]B + \frac{1}{30}HC = 0 \tag{27}$$

$$\frac{1}{30}\gamma HB - \left[\frac{1}{3} + \frac{1}{30}(k^2 + \gamma H)\right]C = 0 \tag{28}$$

Equating the determinant of this system with zero gives an explicit expression for the Rayleigh number

$$R = \frac{a_{11}(a_{22}a_{33} - a_{23}a_{32})}{a_{12}^2 a_{33}} \tag{29}$$

where

$$a_{11} = -\left[\frac{1}{630}Dk^4 + \left(\frac{4}{105}D + \frac{1}{630}\right)k^2 + \frac{4}{5}D + \frac{2}{105}\right]$$

$$a_{12} = \frac{1}{140}k, \qquad a_{22} = -\left(\frac{1}{30}k^2 + \frac{1}{3} + \frac{1}{30}H\right)$$

$$a_{23} = \frac{1}{30}H, \qquad a_{32} = \frac{1}{30}\gamma H$$

$$a_{33} = -\left(\frac{1}{30}k^2 + \frac{1}{3} + \frac{1}{30}\gamma H\right) \tag{30}$$

Minimization of the Rayleigh number over k produces a polynomial equation, which can be solved by routine procedures. To this end, we remark that the explicit form of (29) is

$$R = \frac{28(10 + k^2)(10 + H + \gamma H + k^2)\left[12 + k^2 + D(504 + 24k^2 + k^4)\right]}{24k^2(10 + \gamma H + k^2)} \tag{31}$$

and this explicit expression is especially instructive since it clearly shows that: (i) R is always positive, (ii) $R \to \infty$ as $1/k^2$ when $k \to 0$ and (iii) $R \to \infty$ as k^4 when $k \to \infty$, in a full agreement with Fig. 1 (a)–(d).

3.2. N-terms Galerkin approach

Now the unknown functions f, g and h are expressed as

$$f=\sum_{i=1}^{N}A_i f_i,\qquad g=\sum_{i=1}^{N}B_i g_i,\qquad h=\sum_{i=1}^{N}C_i h_i \tag{32}$$

where the trial functions are taken as

$$f_i=y^{i+1}(1-y)^2,\qquad g_i=y^i(1-y),\qquad h_i=y^i(1-y) \tag{33}$$

After algebraic manipulations, we get the eigenvalue equation

$$\mathbf{MX}=0 \tag{34}$$

where

$$M_{ij}=-D\int_0^1 f_j^{iv} f_i\,\mathrm{d}y+(2k^2D+1)\int_0^1 f_j'' f_i\,\mathrm{d}y - k^2(k^2D+1)\int_0^1 f_j f_i\,\mathrm{d}y$$
$$1\leqslant i\leqslant N,\ 1\leqslant j\leqslant N \tag{35a}$$

$$M_{ij}=k\int_0^1 g_j f_i\,\mathrm{d}y,\quad 1\leqslant i\leqslant N,\ N+1\leqslant j\leqslant 2N \tag{35b}$$

$$M_{ij}=0,\quad 1\leqslant i\leqslant N,\ 2N+1\leqslant j\leqslant 3N \tag{35c}$$

$$M_{ij}=k\int_0^1 f_j g_i\,\mathrm{d}y,\quad N+1\leqslant i\leqslant 2N,\ 1\leqslant j\leqslant N \tag{35d}$$

$$M_{ij}=\int_0^1 g_j'' g_i\,\mathrm{d}y-(k^2+H)\int_0^1 g_j g_i\,\mathrm{d}y$$
$$N+1\leqslant i\leqslant 2N,\ N+1\leqslant j\leqslant 2N \tag{35e}$$

$$M_{ij}=H\int_0^1 h_j g_i\,\mathrm{d}y,\quad N+1\leqslant i\leqslant 2N,\ 2N+1\leqslant j\leqslant 3N \tag{35f}$$

$$M_{ij}=0,\quad 2N+1\leqslant i\leqslant 3N,\ 1\leqslant j\leqslant N \tag{35g}$$

$$M_{ij}=\gamma H\int_0^1 g_j h_i\,\mathrm{d}y,\quad 2N+1\leqslant i\leqslant 3N,\ N+1\leqslant j\leqslant 2N \tag{35h}$$

$$M_{ij}=\int_0^1 h_j'' h_i\,\mathrm{d}y-(k^2+\gamma H)\int_0^1 h_j h_i\,\mathrm{d}y$$
$$2N+1\leqslant i\leqslant 3N,\ 2N+1\leqslant j\leqslant 3N \tag{35i}$$

and $\mathbf{X}=\{A_1,\ldots,A_N,B_1,\ldots,B_N,C_1,\ldots,C_N\}^{\mathrm{T}}$.

Numerical trials showed that $N=10$ terms are enough to get accurate results. Therefore, all the results reported hereinafter are for 10 terms in the Galerkin expansions.

In Table 1 there are listed the critical values of the wave number k_c and Rayleigh number R_c, for 1-term and N-terms Galerkin approaches, when

- $D=0$ (that is for the Darcy flow model), 10^{-3} (which corresponds to a relatively sparse porous medium) and 1. We mention the Darcy number is related to the importance of viscous effects in the region of boundaries, small values of D decreasing this effect, which allows the fluid to move more easily, thereby decreasing the critical Rayleigh number.
- $\gamma=0$, 1 and 10. We remark that low values of γ correspond to a relatively poor conducting fluid (for example, air in a metallic porous medium), while large γ mean that heat is transported through both solid and fluid phases.

The range considered for H extends from very small to very large values (in the limit $H\to\infty$). The local thermal equilibrium is recovered in the large H-limit.

One can easily remark from Table 1 that both k_c and R_c are overestimated by the one-term Galerkin approach for the all range of values considered for H. In this table there are also included the results reported by Banu and Rees [10] when the Darcy number $D=0$, that is for the Darcy flow model. For sake of comparison, available in that paper is a set of results for $\gamma=1$ and H ranging from 0.1 to 10^5. As expected, the agreement between these authors' results and our results is very good for the 10-term Galerkin method. It is easily seen that the agreement is excellent for medium values of H and good for large H.

Fig. 1 shows a selection of neutral curves, R against k/π, for various values of γ and D with $H=100$. We remark the familiar shape for Bénard-like problems, with a single well-defined minimum value and this is the similar trend obtained in Part 1: the values of R_c become smaller as D decreases and as γ increases. Moreover, the minimum values of the Darcy–Rayleigh number are close in the SFB case (see Fig. 2 from Part 1) and in the present case.

In Figs. 2 and 3 we present the behavior of the critical Darcy–Rayleigh number and critical wave number, respectively, as functions of H and γ for $D=0$, $D=10^{-3}$ and $D=1$. Following the same style of graphical representation as in Part 1, in order to facilitate the comparisons, in both Figs. 2 and 3, $\log_{10}H$ is used in the abscissa. We find again essential similarities between the SFB and RB cases, such as:

- R_c increases as H increases and γ decreases.
- When H is small, all the R_c curves corresponding to various γ tend to an asymptotic which does not depend on γ, see Part 1 for additional comments.
- k_c attains the value π for large H, for any γ, except $\gamma=0$, but its maximum values (obtained at intermediate values of H) are larger for RB than for SFB case.
- Critical values of the Darcy–Rayleigh number R_c are larger in the RB case.

Table 1
Comparisons between the results for critical wave number and Darcy–Rayleigh number given by the 1-term (1t) and 10-terms (10t) Galerkin approach

$D = 1$

H	$\gamma = 0$			
	k_c (1t)	R_c (1t)	k_c (10t)	R_c (10t)
10^{-6}	3.120	1795.67	3.120	1752.2105
10^{-5}	3.120	1795.67	3.120	1752.2113
10^{-4}	3.120	1795.68	3.120	1752.219
0.001	3.120	1795.76	3.120	1752.299
0.01	3.120	1796.58	3.121	1753.101
0.1	3.125	1804.76	3.126	1761.109
1	3.171	1885.93	3.173	1840.523
10	3.492	2654.55	3.499	2590.570
100	4.266	9512.66	4.282	9233.510
1000	4.688	75402.0	4.704	72713.77
10^5	4.765	7.31×10^6	4.782	7.034×10^6
	$\gamma = 1$			
10^{-6}	3.11982	1795.67	3.120	1752.2105
10^{-5}	3.11982	1795.67	3.120	1752.2113
10^{-4}	3.11983	1795.68	3.120	1752.2193
0.001	3.11988	1795.76	3.120	1752.2995
0.01	3.12036	1796.58	3.121	1753.1005
0.1	3.12514	1804.72	3.126	1761.0641
1	3.167	1881.67	3.168	1836.336
10	3.307	2387.99	3.311	2330.207
100	3.204	3292.16	3.206	3210.852
1000	3.130	3556.54	3.133	3470.010
10^5	3.120	3590.99	3.120	3504.071
	$\gamma = 10$			
10^{-6}	3.11982	1795.67	3.120	1752.2105
10^{-5}	3.11982	1795.67	3.120	1752.2113
10^{-4}	3.11988	1795.67	3.120	1752.21932
0.001	3.11988	1795.76	3.120	1752.2994
0.01	3.11988	1795.76	3.121	1753.0965
0.1	3.12469	1804.33	3.131	1760.6872
1	3.143	1855.92	3.144	1811.060
10	3.134	1945.59	3.134	1898.395
100	3.122	1971.76	3.122	1924.006
1000	3.120	1974.89	3.120	1927.082
10^5	3.11982	1975.24	3.120	1927.428

$D = 0.001$

H	$\gamma = 0$			
	k_c (1t)	R_c (1t)	k_c (10t)	R_c (10t)
0.001	3.29899	47.2963	3.215	43.151884
0.01	3.29972	47.3167	3.215	43.171098
0.1	3.30696	47.5202	3.223	43.362998
1	3.377	49.5312	3.291	45.259
10	3.906	67.9635	3.812	62.651
100	5.817	212.913	5.686	200.000
1000	8.718	1424.14	8.522	1353.562
10^4	10.271	13091.9	10.032	12481.318
	$\gamma = 1$			
0.001	3.29899	47.2963	3.214	43.1519
0.1	3.29972	47.3166	3.219	43.1711
0.1	3.30689	47.5191	3.222	43.3620
1	3.370	49.4323	3.285	45.163
10	3.610	62.1234	3.514	56.997
100	3.444	86.2664	3.349	78.912
1000	3.316	93.6180	3.230	85.441
10^4	3.301	94.4894	3.216	86.212
10^5	3.299	94.5782	3.215	86.291

Table 1 (*Continued*)

$D = 0.001$

H	$\gamma = 10$			
	k_c (1t)	R_c (1t)	k_c (10t)	R_c (10t)
1	3.335	48.8195	3.249	44.573
10	3.322	51.2040	3.236	46.737
100	3.302	51.9266	3.217	47.379
1000	3.299	52.0316	3.215	47.456
10^4	3.29894	52.0224	3.215	47.464
10^5	3.29891	52.0233	3.214	47.464

$D = 0$

H	$\gamma = 0$			
	k_c (1t)	R_c (1t)	k_c (10t)	R_c (10t)
10^{-5}	3.30975	45.5352	3.162	40.000709
10^{-4}	3.30976	45.5354	3.162	40.000890
0.001	3.30983	45.5373	3.162	40.002702
0.01	3.31058	45.5569	3.163	40.020819
0.1	3.31799	45.7522	3.170	40.201765
1	3.390	47.6811	3.239	41.990
10	3.936	65.3166	3.767	58.379
100	6.028	201.873	5.776	186.738
1000	10.492	1288.19	10.057	1222.361
10^4	18.6167	11112	17.846	10657

	$\gamma = 1$				
H	Present				Banu and Rees [10]
	k_c (1t)	R_c (1t)	k_c (10t)	R_c (10t)	R_c
10^{-5}	3.30975	45.5352	3.162	40.000709	–
10^{-4}	3.30976	45.5354	3.162	40.000890	–
0.001	3.30983	45.5373	3.162	40.002702	–
0.01	3.31058	45.5569	3.163	40.020810	39.498405
0.1	3.31792	45.7512	3.169	40.192163	39.677163
1	3.383	47.5867	3.232	41.897	41.362100
10	3.630	59.7660	3.460	52.991	52.359639
100	3.460	83.0308	3.293	73.258	72.339570
1000	3.328	90.1332	3.178	79.220	78.190834
10^4	3.31156	90.9751	3.164	79.922035	78.879042
10^5	3.30993	91.0608	3.162	79.993431	78.949044

H	$\gamma = 10$			
	k_c (1t)	R_c (1t)	k_c (10t)	R_c (10t)
10^{-5}	3.30975	45.5352	3.162	40.000709
10^{-4}	3.30976	45.5354	3.162	40.000890
0.001	3.30983	45.5373	3.162	40.002701
0.01	3.31057	45.5568	3.163	40.020719
0.1	3.31726	45.7423	3.169	40.192163
1	3.347	47.0004	3.197	41.335
10	3.333	49.2974	3.183	43.336
100	3.313	49.9952	3.165	43.923
1000	3.310	50.0792	3.162	43.993
10^4	3.30978	50.0877	3.162	43.99996
10^5	3.30975	50.0886	3.162	44.000678

4. Conclusion

1. The one-term Galerkin approach proves its utility at least in the initial guessing of the critical wave number and Rayleigh number. There are many papers dealing with convection in clear fluids or in porous media where an one-term Galerkin approach is used solely. On the other hand, some authors prefer a two-

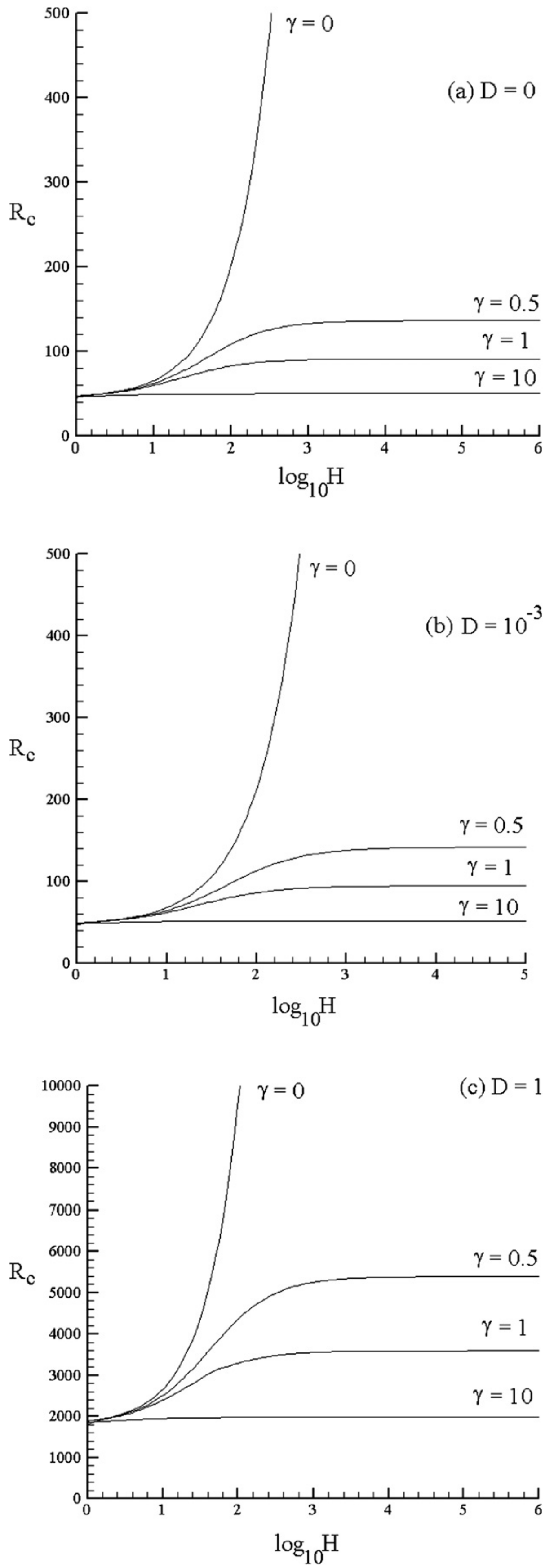

Fig. 2. Variation of the critical Rayleigh number with $\log_{10} H$ for various values of γ: (a) $D = 0$; (b) $D = 0.001$; (c) $D = 1$.

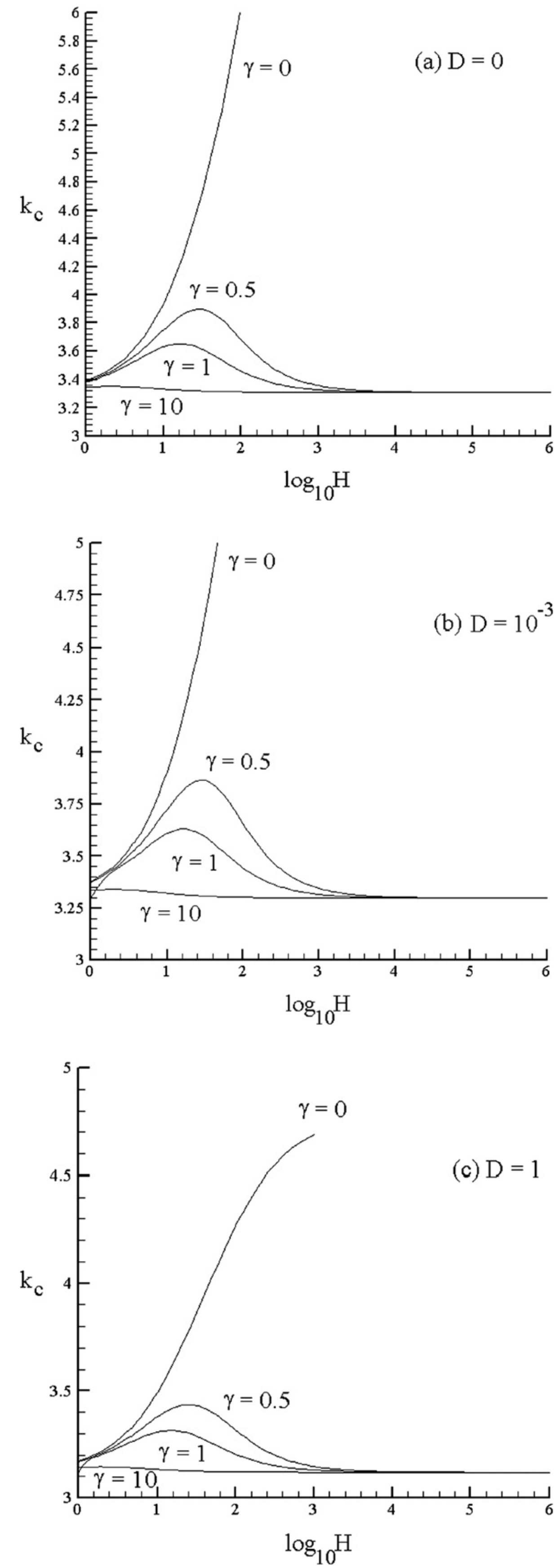

Fig. 3. Variation of the critical wave number k_c with $\log_{10} H$ for various values of γ: (a) $D = 0$; (b) $D = 0.001$; (c) $D = 1$.

terms Galerkin technique, by keeping reasonably the amount of algebra and preserving the advantage of an analytical treatment of the problem.

Once determinant that leads to the eigenvalue equation has the rank N greater than 3, say, a N-term Galerkin approach looses its analytical advantage, but obviously it is preferable in order to get accuracy of the results. Note that analytical approach means not only the finding of the critical Rayleigh number and wave number, but also the minimization of the Rayleigh number over k in the present problem or over several parameters in other situations.

At this end, it is instructive to mention an argument in favor of the Galerkin approximation invoked in a very recent paper by Nield and Kuznetsov [18]: they claim that the Galerkin approximation applied to Rayleigh–Bénard problems will lead to an overestimate of the Rayleigh number by not more than 3% (p. 1214 in that paper).

There are of course many other possibilities to solve the eigenvalue problems arising in convective processes (or in other engineering branches). But whenever an iterative procedure is used, there is necessary to provide an initial guess of the eigenvalue, which can be obtained via the one-term Galerkin approach.

2. Another conclusion drawn from the present study is the qualitative and quantitative comparison between the SFB and RB case: the critical Rayleigh number increases as H increases and γ decreases; when H is small, all the R_c curves corresponding to various γ tend to an asymptotic which does not depend on γ; critical wave number attains the value π for large H, for any γ, except $\gamma = 0$.

Many authors advocate that an analysis of SFB case offers results in qualitative agreement with the RB case. Our paper offers not only a support for this assertion, but gives also quantitative comparisons between these two cases.

Finally, our results show that for Darcy–Bénard convection in a porous layer, in non-equilibrium conditions, both critical wave number and Darcy–Rayleigh number are larger in the RB case than in the SFB case.

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